Homework #5 (100 points) - Show all work on the following problems:

(Grading rubric: Solid attempt = 50% credit, Correct approach but errors = 75% credit, Correct original solution = 100% credit, Copy of online solutions = 0% credit)

Problem 1 (20 points): Derive the exact reflection and transmission coefficients R and T for normal incidence of light on an interface between two materials, without assuming that $\mu_1 = \mu_2 = \mu_0$. Express your answer in terms of $\beta = \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\mu_1 n_2}{\mu_2 n_1}$. Explicitly show that R + T = 1.

Problem 2 (30 points): Construct a graph like the one in Fig. 9.16 for the case of an electromagnetic wave incident from vacuum into diamond, which has an index of refraction n = 2.42. Assume that $\mu_1 = \mu_2 = \mu_0$.

3a (5 points): Calculate the numerical values for the amplitudes E_{0R} and E_{0T} at normal incidence, using the convention of negative amplitude values to indicate if one of the waves is out of phase with the incident wave.

3b (5 points): Calculate Brewster's angle.

3c (10 points): Calculate the crossover angle, where $E_{0R} = E_{0T}$.

3d (10 points): Draw the graph!

Problem 3 (50 points): We have worked exclusively with plane wave solutions so far. However, for point sources of electromagnetic radiation, a more natural solution is a spherical wave. In this case, the real electric field can be written (with $\frac{\omega}{k} = c$):

$$\vec{E}(r,\theta,\phi,t) = A\frac{sin\theta}{r}\left[\cos(kr-\omega t) - \frac{1}{kr}\sin(kr-\omega t)\right]\hat{\phi}$$

For notational convenience, you might wish to write this in the following shorthand form:

$$\vec{E}(r,\theta,\phi,t) = A\frac{sin\theta}{r}\left[\cos u - \frac{1}{kr}\sin u\right]\hat{\phi}$$

If you write it like this, be careful to remember the *r* and *t* dependence of *u* when you take derivatives!

3a (25 points): Plug this electric field into Faraday's law, and integrate with respect to time to find the corresponding magnetic field.

3b (25 points): Calculate the magnitude and direction of the corresponding time-dependent Poynting vector, and then average it over a full cycle of the wave to find the average energy flux (which is the intensity I).